Fundamentals of the Cyclic Short-Time Variation of Indoor Power-line Channels

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Abstract— [1] **In this paper, indoor power-line channels properties are analyzed. The focus is put on the short-term variation that channel exhibits due to the behavior of electrical devices, which is influenced by the presence of mains voltage. Devices high-frequency parameters are time-varying and this makes the channel varies as well. To characterize this behavior it is proposed a model with an LPTV system and cyclostationary noise. In addition, examples of measurements corresponding to actual channels that illustrate these features are shown. More quantitative measurements results performed according to this channel model are presented in [1].**

I. INTRODUCTION

The traditional way to model power-line communications (PLC) channels is by means of a linear time-invariant (LTI) system [2] [3] and a noise composed of two terms: one stationary and other impulsive [4]. In this work, an alternative approach is proposed.

Indoor power lines essentially comprise the wiring and the electrical devices connected to it. The wiring is formed by the different branch circuits that are deployed from the service panel to every outlet and lighting point. In order to establish a communication link, the transmitter and receiver equipment are plugged to any of the outlets or access points through a coupling circuit. The wiring can be modeled as a set of interconnected transmission lines terminated in loads that represent the devices passive behavior [5]. This structural model allows to calculate the channel transfer function, which is going to be closely related to the model considered for the devices.

At the input port of the channel two voltage waveforms are encountered: a large signal term (the one for energy distribution, with 230V and 50Hz in Europe) and a small signal term (the one for communication purposes). The presence of the large signal can make nonlinear signal components to appear at the output of the system, in case it is not completely linear. In fact, although the power grid is a set of transmission lines, formed by pairs of conductors, and they have a quite linear behavior, the electrical devices plugged to the sockets are often not so linear.

On the other hand, noise at the receiver can be analyzed following two criteria. The first one is to classify it depending on the origin, what leads to: a component caused by the appliances plugged in the own power network and another external component originated outside (radio waves coupled to the wires and conducted emissions that comes from the external power distribution network). The second criterion is to organize the noise according to its statistical characteristics. In this case three groups can be established: impulsive noise, stationary noise and cyclostationary noise. In this paper, we are interested only in the latter term of noise, which can include the stationary noise as a particular case and is mainly generated by the electrical devices.

The organization of the paper is as follows. At first the devices behavior is analyzed based on experimental tests, then a slow-variation channel model is proposed. Afterwards, actual channels measurements that confirm the suitability of the model are presented and, finally, some conclusions are given.

II. ELECTRICAL DEVICES BEHAVIOR

It has been verified by means of measurements that many of the electrical devices exhibit a behavior dependent on the mains voltage (230V and 50Hz in Europe). In particular, high-frequency parameters like the impedance towards power network and statistics figures of generated noise, vary synchronously with the mains cycle. This fact leads to discard the usual model of an LTI load that can be represented as a complex frequency-dependent impedance.

The necessity of a better approach can be seen with an experimental test. It consists in measuring the impedance of a coffee-machine, which has been chosen among many other appliances with a similar behavior. In fig.1(a), the real part of the impedance measured with a network analyzer following a conventional procedure in the band from 100kHz to 30MHz is depicted. The curve presents remarkable oscillations due to the fact that the analyzer modifies the sounding frequency slowly and progressively in time. The frequency sweep extends over many mains cycles and the device behavior is changing during that time according to the large voltage variation. As the latter is periodical in time, an oscillation appears in the impedance curve with a period equal to the multiplication of the analyzer sweep-speed by the mains cycle $T_0 = 1/f_0 =$20ms.

The measurement can be processed to obtain different curves for the set of points registered at the same instant, or phase, of mains cycle. This provides a sampled version of the impedance variation at regularly distributed phases over T_0.

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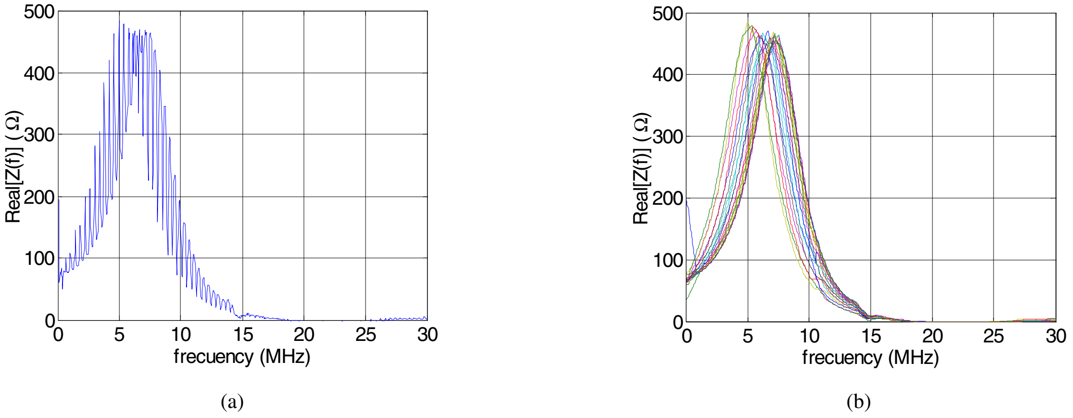


(a)

(b)

Fig. 1. Real part of the impedance measured for a coffee-machine

The procedure result is shown in fig.1(b), where all the curves have been plotted together. The impedance curves are similar to the one of a RLC resonator, but with a resonant frequency that is sensitive to the mains voltage. This experiment supports the necessity of a function of two variables to characterize the impedance of a device, $Z(t, f)$, and so to express the variation in both dimensions, time and frequency. Time evolution can be restricted to T_0, since beyond this point the values are repeated periodically.

It can be inferred that the device cyclic behavior is due to the nonlinear nature of its load. During the test, the device is excited with two superimposed voltage waveforms: the large one at f_0 =50Hz and the small signal generated by the network analyzer. Since the measurement is influenced by the amplitude of the input (approximately the one of mains), it can be concluded that the device under test (DUT) is not linear.

Many measurements of typical appliances have been carried out [7], and two dominant behaviors have been observed: some of them exhibit a gradual variation over mains cycle with a periodicity of 50Hz (like in fig.1(b), also in microwave ovens, monitors, etc.), and others present abrupt changes between two different impedance states with a periodicity of 100Hz (probably caused by SCRs, e.g. in low power lamps, light dimmers...).

In order to complement the preceding experiment, noise generated by the appliances should be analyzed as well. The purpose is to verify if noise statistical parameters also manifest a time variation. To do so, a measurement setup with a spectrum analyzer can be employed, although the analyzer sweep must be triggered with mains voltage. The DUT must be supplied with mains, but using a low pass filter that attenuates the noise that comes from the power network in the band of interest. With this arrangement, it is possible to get a periodogram of the noise generated by the DUT at a certain phase of the mains cycle (which corresponds to a certain mains voltage level). The noise power spectral density (PSD) can be estimated by averaging many of these periodograms.

As an example, in fig.2 the estimated noise PSD for a low power lamp is presented. The two curves have been registered triggering the analyzer at two different phases of the cycle (one corresponds to 0V and the other to 205Vrms). The result in both cases is quite different and it would be also different from the one obtained by a conventional procedure using a self-triggered mode in the analyzer.

This test confirms that the random process is not stationary. In contrast, it should be characterized as a cyclostationary process with a periodical PSD, $S_N(t, f)$, which is properly called its instantaneous PSD [6].

Hence, the proposed model for the devices has two time-varying parameters: impedance $Z(t, f)$ and noise instantaneous PSD $S_N(t, f)$. Additionally, the time variation is considered to be slow enough so that they can be computed by means of a sequence of states, each of them with a PSD of stationary noise and an invariant frequency-dependent impedance, which appears periodically in subsequent mains cycles.

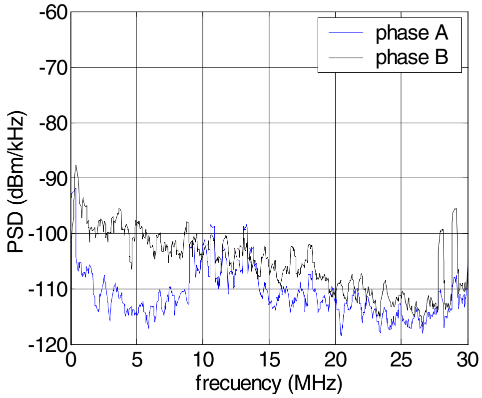


Fig. 2. Noise PSD measurement result of a low power lamp

III. Cyclic channel model

In this section, a model for PLC channels grounded on the previous devices characterization is proposed. First, the channel response modeling procedure is analyzed and afterwards the one for the noise.

A. Channel response

As mentioned, PLC channels can be studied as a set of multiple interconnected transmission lines that contain loads of devices with a nonlinear behavior. Hence, the more realistic behavioral model for these channels is a nonlinear system. However, the great separation, both in level and frequency, of the large and small signal components at the input of the system, permits to use a simpler model. The nonlinear effects created by the system depend only on mains level, because small signal level is irrelevant in this aspect. Moreover, most of nonlinear terms at low frequencies (multiples of f_0) are filtered by the coupling circuit at the receiver. Under these circumstances, it is possible to consider that, from the small signal point of view, the system is linear but periodically time-varying (LPTV) synchronously with mains [7].

A linear time-varying system can be described by means of the input-output relation,

$$y(t) = \int_{-\infty}^{+\infty} h(t, t-\tau) x(t-\tau) d\tau \tag{1}$$

Where $h(t, t-\tau)$ is the system response to an impulse measured at t but applied τ seconds before [8]. For LPTV systems, this impulse response is periodical in time with a period T_0, that is,

$$h(t, t-\tau) = h(t - nT_0, t - nT_0 - \tau) \tag{2}$$

The frequency response of the system is the Fourier transform of the impulse response in the variable τ,

$$H(t, f) = \int_{-\infty}^{+\infty} h(t, t-\tau) e^{-j2\pi f\tau} d\tau \tag{3}$$

Which is also a periodical function in t with a period T_0, and thus, can be expanded in Fourier series as follows,

$$H^{\alpha}(f) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} H(t, f) e^{-j2\pi\alpha t/T_0} dt \tag{4}$$

It can be proved that the relation between the input signal $x(t)$ and the output signal $y(t)$ of an LPTV system in the frequency domain is [7],

$$Y(f) = \sum_{\alpha=-\infty}^{+\infty} H^{\alpha}\left(f - \frac{\alpha}{T_0}\right) X\left(f - \frac{\alpha}{T_0}\right) \tag{5}$$

However, this LPTV formulation can be simplified for PLC channels because their time variation is quite slow and their period is very long. Measurements reveal that PLC channels coherence time, understood as the interval in which channel properties can be considered invariant, is several orders of magnitude above the effective length of the channel impulse response (usually measured by the delay spread). In other words, they can be seen as underspread channels, and the channel response can be calculated from a sequence of states characterized with an LTI response, which appear periodically in time.

Let us denote by $x_\sigma(t)$ a short-time input signal, shorter than the channel coherence time and applied at $t \approx \sigma$ (i.e. an interval around σ). Then the channel output to this signal is,

$$y_\sigma(t) = \int_{-\infty}^{+\infty} h(t, t-\tau) x_\sigma(t-\tau) d\tau \simeq \int_{-\infty}^{+\infty} h_\sigma(\tau) x_\sigma(t-\tau) d\tau \tag{6}$$

As the channel impulse response does not change substantially in $t \approx \sigma$, it has been substituted by $h_\sigma(\tau) = h(t, t-\tau)|_{t=\sigma}$, the LTI response measured in this interval. In the frequency domain the relation can be express as,

$$Y_\sigma(f) \simeq H(t, f)|_{t=\sigma} \cdot X_\sigma(f) \tag{7}$$

This idea can be generalized to any sort of input signal, because a longer signal can be represented by means of a set of short-time signals,

$$x(t) = \sum_{\sigma \in S} x_\sigma(t) \tag{8}$$

Each in a time interval σ within the set S of intervals whose union expands the duration of $x(t)$. Since the channel is assumed linear, the channel output is the superposition of the short-time output signals,

$$y(t) = \sum_{\sigma \in S} y_\sigma(t) \tag{9}$$

Because of the proposed slow-variation approach, PLC channels can be studied by a sequence of LTI systems. These can be characterized as a set of transmission lines terminated in a linear load with a certain frequency-dependent impedance, which corresponds to a locally approximated value of its time-varying impedance. Hence, the channel response can be calculated by a conventional analysis of transmission lines [5].

B. Received noise

Maybe the most important source of noise in PLC channels are the electrical devices, and especially the ones in the own power network of the receiver. As stated in section II, the most appropriate model for this noise is to consider it as a cyclostationary random process. A process $X(t)$ is wide-sense cyclostationary if both its mean and autocorrelation function are time-dependent and periodical in t with a period T_0. The autocorrelation Fourier transform, usually referred to as the instantaneous PSD is [6],

$$S_X(t, f) = \int_{-\infty}^{+\infty} R_X(t, t+u) e^{-j2\pi f u} du \tag{10}$$

This function is also periodical in t and can be expanded by a Fourier series, whose first coefficient is the time-average over T_0 and named the PSD of the cyclostationary process,

$$S_X^0(f) = < S_X(t, f) > = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} S_X(t, f) dt \tag{11}$$

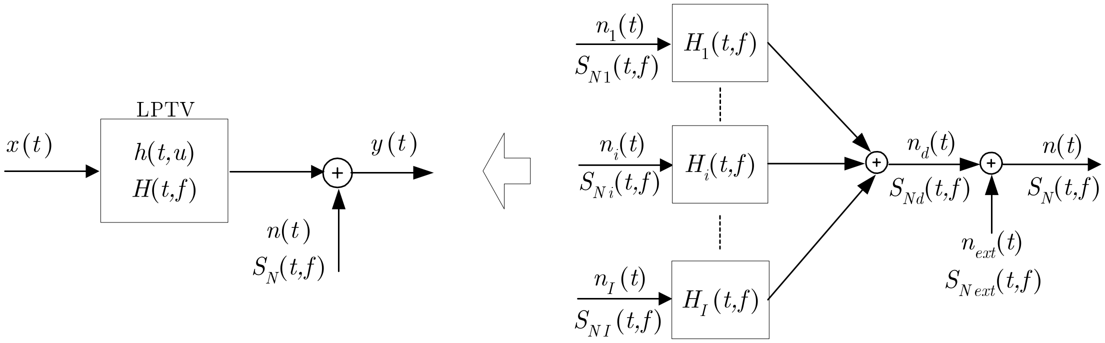


Fig. 3. Diagram of the proposed cyclic channel model

When a cyclostationary signal is filtered by an LPTV channel some spectral broadening appears due to the channel time variation but, under slow-variation conditions, the output signal instantaneous PSD can be calculated by [7],

$$S_Y(t,f) \simeq |H(t,f)|^2 \cdot S_X(t,f) \tag{12}$$

This expression holds true whenever the autocorrelation of the input signal is 'shorter' than channel coherence time. From (12), it is inferred that even if the input signal is stationary, the output will be cyclostationary because of the channel filtering process.

In PLC channels, the noise that reaches the receiver can be separated in two groups: noise from appliances connected to the own power network, $S_{Nd}(t,f)$, and an external noise, $S_{Next}(t,f)$. The former is conducted by the power lines and so it is filtered by the effective channel between its outlet and the receiver one; the latter does not have a clear origin and can be directly added at the receiver. Hence, if a number of I independent noisy devices is assumed in the power-network, the instantaneous PSD of the total received noise can be expressed as,

$$S_N(t,f) = S_{Nd}(t,f) + S_{Next}(t,f) \tag{13}$$

where the first component is,

$$S_{Nd}(t,f) = \sum_{i=1}^{I} S_{Ni}(t,f) \cdot |H_i(t,f)|^2 \tag{14}$$

C. Channel model proposal

The proposed cyclic channel model consists of a slow variation LPTV channel, which represents the propagation path between the transmitter and receiver outlets, and an additive cyclostationary noise. In fig.3, a diagram of this model is depicted. For completeness, the model should incorporate some additive sporadic impulsive noise (without any periodicity with respect to mains cycle), whose impulses interarrival time is several orders of magnitude above channel coherence time [4].

IV. ILLUSTRATIVE EXAMPLES OF ACTUAL CHANNELS

In this section, some example both of channel frequency response and received noise instantaneous PSD are presented. They have been selected from the results of many measurements performed on actual channels in diverse scenarios (detached house, apartment and university building) [1], [7], and can be considered representative of the whole set.

At first, it is shown if fig.4 the magnitude of the frequency response of a channel measured at an apartment between outlets of different branch circuits (of about $80m^2$ and four branch circuits). The link distance would be about 25m for the main path of transmission line. The frequency range measured is from 1MHz to 20MHz, limited by the coupling circuit used. A clear periodicity of 100Hz is observed, especially in the bands around 2MHz and 6.5MHz.

To remark the channel time variation two additional graphs have been included. In fig.5(a), the evolution over the mains cycle of the response at two frequencies is plotted: 1.56MHz in dashed line and 2.64MHz in solid line. There is an excursion higher than 7dB at 2.64MHz with a shape repeated every 10ms and quite different from the shape at 1.56MHz. However, this figure shows only the magnitude and it is maybe more interesting to observe the evolution of the response in the complex plane, as depicted in fig.5(b) (values have been normalized for clarity). Every point in this graph corresponds to an interval in which mains cycle has been divided (shorter than coherence time) to sample the time variation. It should

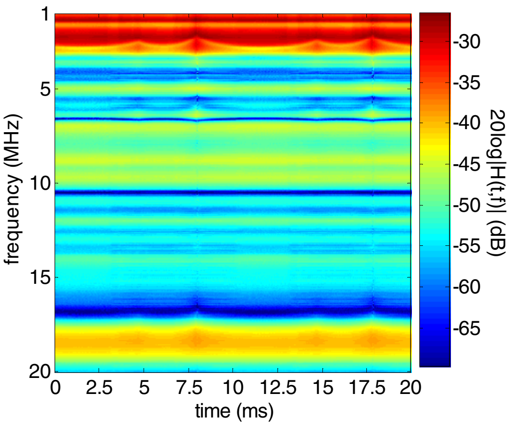
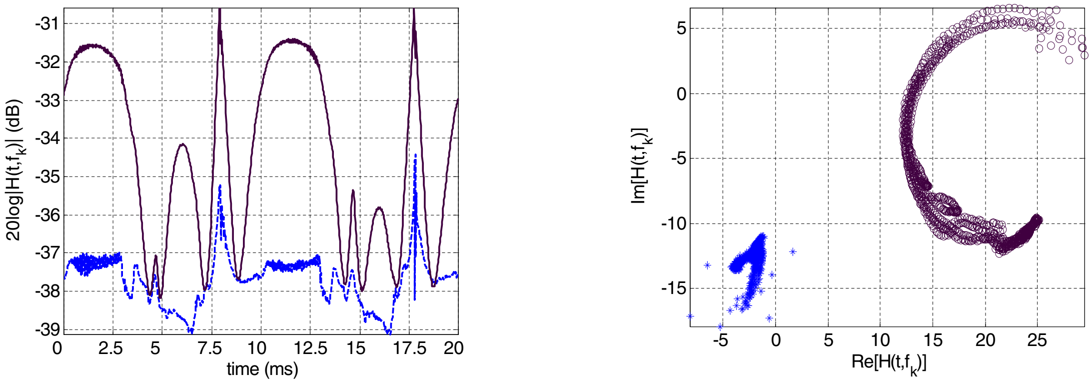


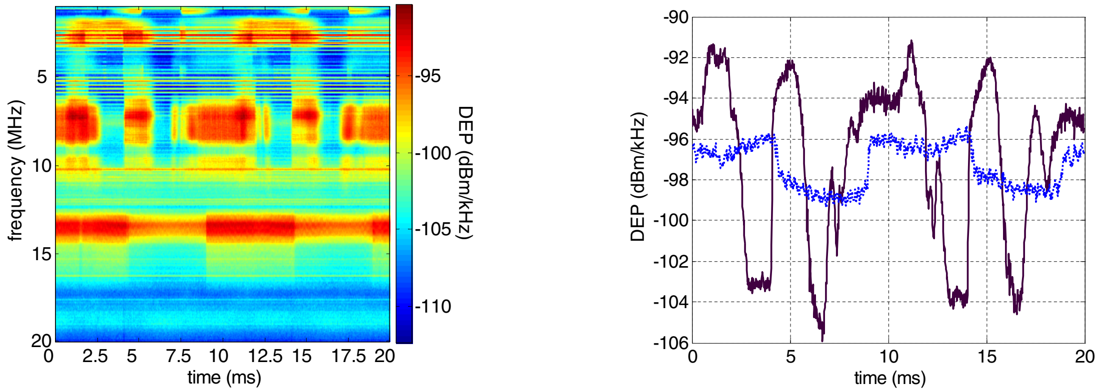
Fig. 4. Magnitude of the frequency response of a channel measured in an apartment



(a)

(b)

Fig. 5. Time evolution of the channel response in fig.4 at two frequencies: a) magnitude at 1.56MHz (blue dashed) and 2.64MHz (black solid); b) complex response at 1.56MHz (blue '*') and 2.64MHz (black 'o').



(a)

(b)

Fig. 6. In a) Instantaneous PSD of the noise measured for a channel at a university building. In b), time evolution of this noise at two frequencies: 13MHz in blue dotted line and 7.3MHz in black solid line.

be mentioned how the phase of the response is progressively changing during the cycle.

Regarding the received noise, another measurement result is presented. In this case, it has been taken from a university laboratory. In fig.6(a), is provided an estimate of the noise instantaneous PSD at a certain outlet that represents a spectrogram over mains cycle. It exhibits many different areas with important time-variation in the measured frequency range. To give a better insight, two curves have been plotted in fig.6(b), extracted from the previous one, which correspond to the noise evolution in time at two different frequencies. The excursion of the noise exceeds 10dB and the periodicity is again 100Hz.

V. CONCLUSION

In this paper, the high-frequency properties of electrical devices connected to indoor power-line networks have been studied. Experimental results that reveal a time-varying behavior have been provided. This fact supports a cyclic channel model proposal whose parameters are periodical in time. Examples of measurements in actual PLC channels have confirmed this behavior, which must be considered to design efficient transmission systems. For instance, if linear modulation schemes are used at many frequencies, the time variation of the channel response should be compensated, otherwise the receiver performance would be degraded.

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